Equations (1) and (4) are substituted in equation (5) and starting with nominal values of Re and δ^+, equations (5) and (6) are solved iteratively. The corresponding heat transfer coefficient is obtained by integrating the energy conservation equation;

$$\frac{h_c}{k}\left(\frac{v^2}{g}\right)^{\frac{1}{3}} = \frac{Pr(\delta^+)^{\frac{1}{3}}}{F(\delta^+)}, \tag{7}$$

where

$$F(\delta^+) = \int_0^{\delta^+} \frac{dy^+}{\frac{1}{Pr} + \frac{1}{Pr_t}(\varepsilon_M^+ - 1)}. \tag{8}$$

Figure 1 compares our predictions with the Chun–Seban experiments. Data was obtained at four saturation temperatures: 28, 38, 62 and 100°C; the corresponding liquid Prandtl numbers were 5·7, 5·1, 2·91 and 1·77 respectively. The agreement is seen to be excellent. At 28°C we are in effect demonstrating the consistency of the experimental data on which our calculation procedure is based. The good agreement at other temperatures confirms that the scaling with surface tension in equation (3) is appropriate. We note further that (i) y_i^+ varied from 0·3 δ^+ to 0·85 δ^+ as Re increased through the range considered, and (ii) the gas absorption experiments showed the mass transfer coefficient to be proportional to $Re^{0.8}$; thus the result $h_c \propto Re^{0.4}$ is not unexpected despite the fact that the near wall transport model suggests $h_c \propto Re^{0.2}$. Figure 1 of course applies to turbulent film condensation as well. Extension of this work to include the effects of vapor drag must await the availability of appropriate gas absorption experimental data.

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JOULE–THOMSON EFFECTS ON THERMAL ENTRANCE REGION HEAT TRANSFER IN PIPES WITH UNIFORM WALL TEMPERATURE

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NOMENCLATURE

Br, Brinkman number, $\mu U_m^2/k(T_0 - T_w) = PrEc$;
c_p, specific heat at constant pressure;
$h, \bar{h}$, specific enthalpy and local heat transfer coefficient, respectively;
k, thermal conductivity;
p, local axial pressure;
$\bar{q}$, heat flux vector;
R, X, radial and axial coordinates;
R_0, pipe radius;
T, T_0, T_w, gas temperature, uniform gas temperature at thermal entrance and constant wall temperature, respectively;

U, U_m, gas velocity and mean gas velocity in X-direction;
μ, μ_J, absolute viscosity and Joule–Thomson coefficient, $(\partial T/\partial p)_h$;
ρ, gas density;
Φ, viscous dissipation function.

1. INTRODUCTION

It is known that the Joule–Thomson effects are important only at low enough temperatures and high enough pressures and a positive or negative Joule–Thomson coefficient results from the departure of real gases from the perfect-gas behaviour. For many engineering applications, the Joule-Thomson effect may be negligible but for applications such as arctic gas pipeline its effect becomes of primary importance. As a matter of fact, the Joule–Thomson effect was confirmed in the experimental investigations carried out by a consortium of oil industries in the arctic test facilities using air as test gas.

The purpose of this paper is to study the Joule–Thomson effects on thermal entrance region heat transfer for fully-developed laminar gas flows in pipes with uniform wall temperature.

2. THEORETICAL ANALYSIS

The assumptions and formulation of the Graetz problem with uniform wall temperature [1] are well-known and need not be repeated here. For gas flows in pipes, the pressure work is of the same order as the viscous dissipation and both effects should be accounted for in the analysis. The energy equation in terms of enthalpy [2] may be written as

$$\rho \frac{Dh}{Dt} = -\nabla \cdot \bar{q} + \frac{Dp}{Dt} + \mu\Phi. \tag{1}$$

The thermodynamic equation for the difference in specific enthalpy between two neighboring equilibrium states in terms of the Joule–Thomson coefficient is [3]

$$dh = c_p\,dT - \mu_J c_p\,dp \quad \text{where} \quad \mu_J = (\partial T/\partial p)_h.$$

For constant-property gas and noting that all thermodynamic functions such as temperature, enthalpy, etc., may retain their classical meaning even though the gas is in motion, then

$$\frac{Dh}{Dt} = c_p \frac{DT}{Dt} - \mu_J c_p \frac{Dp}{Dt}. \tag{2}$$

Substitution of equation (2) into (1) yields

$$\rho c_p \frac{DT}{Dt} = k\nabla^2 T + (1 + \rho c_p \mu_J)\frac{Dp}{Dt} + \mu\Phi. \tag{3}$$

For the Graetz problem dealing with the steady fully-developed laminar gas flow, the energy equation in terms of cylindrical coordinates becomes

$$\rho c_p U \frac{\partial T}{\partial X} = k \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) + (1 + \rho c_p \mu_J) U \frac{dp}{dX} + \mu\left(\frac{\partial U}{\partial R}\right)^2. \tag{4}$$

It is convenient to render equation (4) dimensionless by use of the following definitions:

$$r = R/R_0,\ x = X/R_0 RePr,\ u = U/(2U_m) = (1 - r^2),\quad \theta = (T - T_w)/(T_0 - T_w),$$

$$Re = \rho(2R_0U_m)/\mu,\ Pr = c_p\mu/k,\ Ec = U_m^2/c_p(T_0 - T_w),$$

$$Eu_t = [(T_0 - T_w)/\mu_J]/\rho U_m^2 \quad \text{where} \quad U_m = -(R_0^2/8\mu)(dp/dX).$$

Thus equation (4) becomes

$$u\frac{\partial\theta}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) - 16\left(PrEc + \frac{Pr}{Eu_t}\right)u + 4PrEc\left(\frac{\partial u}{\partial r}\right)^2 \tag{5}$$

with boundary conditions

$$\theta(0, r) = 1,\ \theta(x, 1) = 0,\ \partial\theta(x, 0)/\partial r = 0. \tag{6}$$

In view of the definition for Joule–Thomson coefficient $\mu_J = (\partial T/\partial p)_h$, the new characteristic parameter Eu_t corresponding to Euler number in flow problem appears in the present thermal convection problem and it may be called "thermal Euler number". It is seen that the pressure drop in Euler number is simply replaced here by the equivalent pressure drop $(T_0 - T_w)/\mu_J$ corresponding to the temperature change $(T_0 - T_w)$. It is noteworthy that the thermal Euler number is also related to Eckert number by the relationship $(1/Eu_t) = (\rho c_p \mu_J)Ec = (\partial T/\partial p)_h \cdot \rho U_m^2/(T_0 - T_w)$. Depending on the sign of $(T_0 - T_w)$, both Eckert number and thermal Euler number can be positive (cooling) or negative (heating). It can be seen from equation (4) that viscous dissipation acts as a distributed heat source and pressure work becomes a distributed heat sink. Also, depending on the sign of Joule–Thomson coefficient μ_J, the term involving μ_J acts as a distributed heat sink (μ_J = positive) or source (μ_J = negative).

Excluding the region near the entrance, the problem is most conveniently solved by the Graetz method [4] using superposition theory. It can be shown that the solution is obtained in the following form.

$$\theta = \sum_{n=1}^{\infty}\left(C_n + PrEcK_n + \frac{Pr}{Eu_t}L_n\right)Y_n(r)\exp(-\lambda_n^2 x) - (1 - r^2)\left[(3 - r^2)\frac{Pr}{Eu_t} + 2(1 - r^2)PrEc\right] \tag{7}$$

where $C_n = -2/\lambda_n(\partial Y_n/\partial\lambda_n)_{r=1}$, $K_n = -32/\lambda_n^3(\partial Y_n/\partial\lambda_n)_{r=1} - (32/\lambda_n)\int_0^1 r^3 Y_n(r)\,dr/(\partial Y_n/\partial r \cdot \partial Y_n/\partial\lambda_n)_{r=1}$, $L_n = -32/\lambda_n^3(\partial Y_n/\partial\lambda_n)_{r=1}$.

The eigenvalues λ_n^2 and eigenfunctions $Y_n(r)$ satisfy the Sturm–Liouville system [5]. Using the first eleven values for λ_n, $(\partial Y_n/\partial \lambda_n)_{r=1}$ and $(\partial Y_n/\partial r)_{r=1}$ listed by Brown [5], the coefficients C_n, K_n and L_n are computed and the values are given in Table 1.

The bulk temperature and Nusselt number are of interest in design.

$$\theta_b = 4\int_0^1 \theta\, ur\, dr = -4\sum_{n=1}^{\infty} C_n + PrEcK_n + \frac{Pr}{Eu_t} L_n \quad (1/\lambda_n^2).$$

$$dY_n(1)/dr \cdot \exp(-\lambda_n^2 x) - PrEc - (11/6)(Pr/Eu_t) \quad (8)$$

$$Nu = (2R_0\bar{h}/k) = -2(\partial\theta/\partial r)_{r=1}/\theta_b. \quad (9)$$

For long pipes, one is particularly interested in the following asymptotic results.

$$\theta_{bf} = -[PrEc + (11/6)Pr/Eu_t] = -PrEc[1 + (11/6)\rho c_p \mu_J] \quad (10)$$

$$Nu_f = (8Pr/Eu_t)/[PrEc + (11/6)(Pr/Eu_t)] = 8\rho c_p \mu_J/[1 + (11/6)\rho c_p \mu_J]. \quad (11)$$

The three limiting cases of the above general results are of special interest. The classical Graetz solution is obtained by setting $Ec \to 0$ and $Eu_t \to \infty$ (or $\mu_J \to 0$) with the limiting Nusselt number of 3·66 obtained by considering the first term of the series. For Brinkman problem [6] applicable to liquid only, the term involving $U dp/dX$ in equation (4) vanishes with $\rho c_p \mu_J = -1$. Thus the asymptotic Nusselt number is 9·6. For gases with pressure work and viscous dissipation effects but without Joule–Thomson effect ($\mu_J = 0$), one simply obtains $Nu_f = 0$. It is interesting to note that the bulk temperature θ_{bf} vanishes when $\rho c_p \mu_J = -6/11$ and positive and negative Joule–Thomson coefficients have different effect on bulk temperature.

3. RESULTS

Because of space limitation, only Nusselt number results for representative parametric values will be presented here. The Joule–Thomson effects on local Nusselt number variations are shown in Fig. 1 for $PrEc = 0·1$ and 1·0 (cooling) and in Fig. 2 for $PrEc = -0·1$ and $-1·0$ (heating) with Pr/Eu_t or $(\rho c_p \mu_J)$ $PrEc$ as parameter. It is noted that for nitrogen, for example, the value of the parameter $\rho c_p \mu_J$ may vary approximately from 0·01 to 0·9 for gas pressure ranging from 0 to 800 psia. The decrease of Nusselt number with axial distance such as that in Leveque solution region of the Graetz problem is known to be entrance effect.

In Fig. 1, one notes that a minimum Nusselt number appears at some axial distance and from that point onward the Nusselt number increases until a singularity corresponding to zero bulk temperature ($\theta_b = 0$) is approached. After passing the singularity the Nusselt number becomes zero ($\partial\theta/\partial r = 0$) at a further downstream position signifying the change of direction of heat transfer at wall. This is apparently caused by cooling effect due to internal distributed heat sinks. Without Joule–Thomson effect, the asymptotic Nusselt number is zero but with Joule–Thomson effect the asymptotic Nusselt number is given by equation (11). Physically, the minimum Nusselt number represents the balance between the entrance effect and the combined pressure work, viscous dissipation and Joule–Thomson effects and after occurrence the combined three effects dominate. This observation is also confirmed by the fact that up to the point of minimum Nusselt number the Joule–Thomson effect is seen to be relatively insignificant. In contrast, the Joule–Thomson effect becomes significant as the asymptotic condition is approached. The different trends in Nusselt number behaviour shown in the inset, for example, at $x = 5 \times 10^{-3}$ and 10^{-2} can be traced to the relative magnitudes of $(\partial\theta/\partial r)_{r=1}$ and θ_b in the definition for Nusselt number.

The Joule–Thomson effect as an additional cooling effect is clearly seen in Fig. 2 for heating case. With $PrEc = -0·1$ the local Nusselt number decreases monotonically from entrance value to an asymptotic value. Noting that the asymptotic Nusselt number is independent of $PrEc$, the minimum Nusselt number appears with $PrEc = -1·0$.

It should be pointed out that for application in arctic natural gas pipeline the flow is turbulent. In view of the difficulty in solving turbulent heat convection problem with Joule–Thomson effects, the present result on laminar flow can provide a useful guide in correlating experimental data for turbulent flow with Joule–Thomson effects.

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Table 1. Coefficients C_n, K_n *and* L_n

C_n:	1·476435 ($n=1$)	−0·806124	0·588762	−0·475850
	0·405022	−0·355757	0·319169	−0·290736
	0·267891	−0·249063	0·233228 ($n=11$)	
K_n:	1·876600 ($n=1$)	0·214896	−0·149489	0·0969863
	−0·0667903	0·0485528	−0·0368237	0·0288679
	−0·0232336	0·0191012	−0·0159816 ($n=11$)	
L_n:	3·230011 ($n=1$)	−0·289131	0·0826906	−0·0353725
	0·0185916	−0·0110765	0·00718023	−0·00494582
	0·00356631	−0·00266519	0·00204975 ($n=11$)	

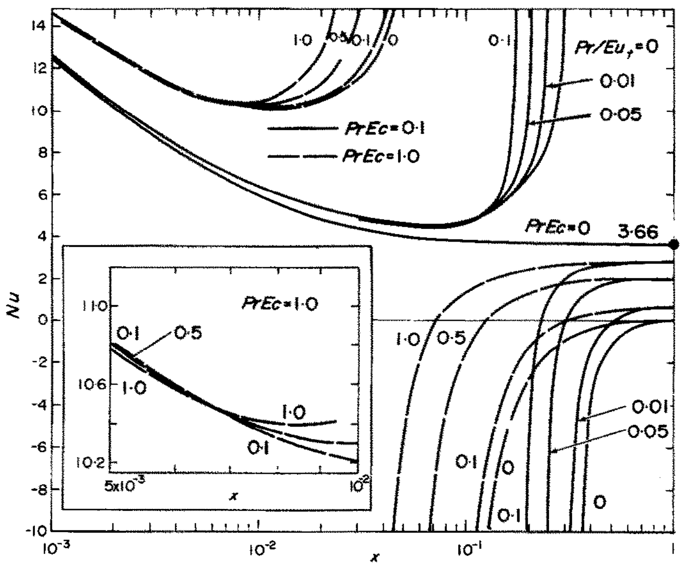

FIG. 1. Local Nusselt number results for $PrEc = 0{\cdot}1$ and $1{\cdot}0$ with Pr/Eu_t as parameter.

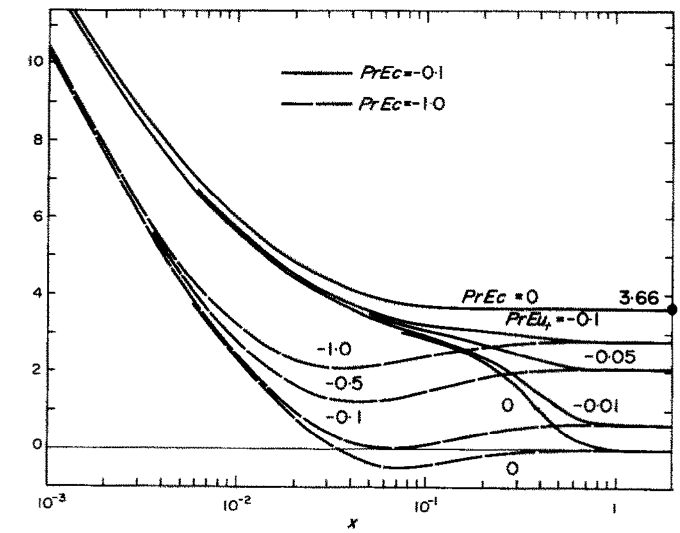

FIG. 2. Local Nusselt number results for $PrEc = -0{\cdot}1$ and $-1{\cdot}0$ with Pr/Eu_t as parameter.

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